

Possible solution to basic problems regarding the coupling constants G_V and G_A

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Abstract

The existing experimental evidence on the vector and axial-vector coupling constants, G_V and G_A , for neutron β -decay exhibits two prominent problems: (i) the unitarity for the first row of the CKM matrix seems violated; (ii) one obtains different values of G_A/G_V according to whether one uses as input the neutron lifetime or neutron-decay correlation observables. We show that the in-medium modification of G_A can influence both the inner and outer radiative corrections used in deducing G_V from the super-allowed Fermi transitions and that this effect may resolve the long-standing problems (i) and (ii) simultaneously.

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The vector coupling constant G_V and the axial-vector coupling constant G_A for neutron β -decay are quantities of primary importance that have been studied extensively. The recent remarkable progress in the determination of G_V and G_A highlights two very notable problems. One problem is concerned with G_V and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ud} . Comparison of G_V with G_μ obtained from the μ -decay rate gives the empirical value of V_{ud} , and the existing data give [1]

$$V_{ud} = 0.9740 \pm 0.0005. \quad (1)$$

Since in the standard model the CKM matrix is unitary, the first row of the CKM matrix must satisfy $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. However, V_{ud} in Eq.(1) combined with the available information on V_{us} and V_{ub} leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9972 \pm 0.0013, \quad (2)$$

indicating a violation of unitarity at the 98% confidence level [1]. Since this violation, if confirmed, constitutes a clear signal for physics beyond the standard model, and since the contribution of V_{ud} dominates the sum in Eq.(2), it is vitally important to assess the reliability of V_{ud} in Eq.(1).

The second problem concerns $\lambda \equiv G_A/G_V$. The neutron-decay angular distribution experiments give [2]

$$\lambda_A = 1.2601 \pm 0.0025, \quad (3)$$

whereas the neutron lifetime measurements, which have recently greatly improved, give [2]

$$\lambda_\tau = 1.2681 \pm 0.0033. \quad (4)$$

(For the neutron lifetime we use the value 888 ± 3 sec which is between the two recommended values 887 ± 2.0 sec and 889.2 ± 2.2 sec.) The origin of this discrepancy in λ is at present unknown, a situation that is rather disturbing.

In this Letter we point out the possibility that the above two problems are related to each other and that they may be solved simultaneously by taking into account certain types

of hadronic effects which have not been considered so far. As is well known, G_V is obtained from the observed ft values for $0^+ \rightarrow 0^+$ nuclear β -decays after applying radiative corrections and isospin-mixing corrections. Although these corrections have been studied in great detail by many authors, we present here a new type of nuclear correction which increases G_V , and hence V_{ud} as well, in such a way that the unitarity condition is satisfied. This increase in G_V decreases the value of λ_τ due to the fact that the neutron decay rate is proportional to $G_V^2 + 3G_A^2 = G_V^2(1 + 3\lambda_\tau^2)$. By contrast, λ_A remains unaffected, because it is directly related to the correlation observables in neutron β -decay and hence has no connection with the ft values of the $0^+ \rightarrow 0^+$ transitions. We will argue that our new value of λ_τ is compatible with λ_A .

In deducing the empirical value of V_{ud} from comparison of the rates of $0^+ \rightarrow 0^+$ nuclear β -decays and the strength of the pure leptonic μ -decay, an accurate estimate of radiative corrections is the most important step (see for example, [1,3–5] and references therein). The bench mark work which constitutes the basis of all the recent analyses is due to Marciano and Sirlin [4], who find the following corrections

$$\Delta_{O(\alpha)} = \frac{\alpha}{2\pi} [\bar{g}(E_m) + 4 \ln(m_Z/M) + \ln(M/M_A) + 2C + A_g]. \quad (5)$$

The first term, often called the outer correction, represents a spectrum-average effect which depends on the β -ray end-point energy E_m , and its contribution amounts to ~ 0.01 [6]. Of the remaining terms (inner corrections), the $4 \ln(m_Z/M)$ term represents the dominant model-independent short-distance contribution (M = proton mass, m_Z = Z-boson mass), whereas $\ln(M/M_A) + 2C$ are axial-current induced contributions. M_A (~ 1 GeV) is a low-energy cutoff characterizing the short-distance part of the γW box diagram, while C represents the long-distance correction. Numerically, the contributions of the short-distance and long-distance parts are comparable, each contributing about 0.001 (see, e.g. ref. [9]). Finally, A_g is a perturbative QCD correction, which turns out to be very small. The standard radiative corrections above have been obtained for the weak quark current $\bar{q}\gamma^\mu(1 + \gamma_5)q$. In applying these results to nucleon and nuclear β -decays, one needs to translate the quark-

based description to hadronic descriptions, and this step is considered to introduce a certain amount of model dependence.

The fact that the axial current can contribute to the radiative correction of the Fermi transition means that, if one uses the nucleon weak current instead of the quark current, the results will in general depend on λ , the strength of the axial-current coupling, and that the well-established nuclear-medium dependence of λ is capable of affecting the radiative correction. In fact, parts of such effects have been studied in the literature. Several authors [6–8] investigated the two-nucleon contribution to the C term in Eq. (5), while Towner [9] estimated the influence of the in-medium modification of λ on the C term. Changes in C due to these effects were reported to be small. The existing work is limited to the C term because in the treatment of Ref. [4], the $\ln(M/M_A)$ term, representing the short-distance effects was considered to be independent of hadron/nuclear models, leaving only the C term as a model-dependent contribution. We note, however, that, although the separation of the axial-vector induced corrections into short- and long-distance parts has a clear-cut meaning in the quark picture, there is a possibility that this separation becomes less well defined when one switches (as one must do at some point) to hadronic descriptions.

In this note we estimate the effects of the in-medium modification of λ , using the nucleonic weak current from the outset, and without invoking separation into short- and long-distance contributions. This means that our calculation in principle incorporates the combined effects of the $\ln(M/M_A)$ and C terms, even though we do not try to make direct contact with the quark-model-based classification used in Eq. (5). As will be demonstrated below, our approach indeed gives rise to a cut-off dependent term which is a function of λ and hence can be influenced by nuclear-medium effects. It is important to emphasize, however, that we work here with the nucleonic weak current instead of the quark weak current *not* because we think the former has more fundamental meaning. Rather, the reason is that the nucleonic current description provides an interesting alternative framework for incorporating the nuclear medium effects.

Our work has some overlap with that of Towner [9] in that the effects of in-medium

modification of λ are considered but, as mentioned above, the calculation in [9] was limited to the C term. Obviously, a calculation of the entire λ -dependent effect (rather than just the C term) is an extremely difficult task. In the present exploratory work, therefore, we present an estimate based on schematic models. Thus, we use the nucleon weak current

$$\bar{u}_p \gamma^\mu (1 + \lambda \gamma_5) u_n, \quad (6)$$

and assume that the nuclear-medium effect can be represented by changing the free space value λ into an in-medium value λ^* . As for electromagnetic interactions we take into account the nucleon charge and magnetic moments and the electron charge. To calculate the loop diagrams, we employ two simplified approaches. In the first approach we use a single common cut-off parameter Λ_0 instead of form factors at the vertices. In the second approach we use a common form factor $\Lambda^2/(q^2 + \Lambda^2)$, both at the weak and at the electromagnetic vertices, where q is transferred momentum, and Λ is the parameter of the same order of magnitude as the nucleon mass M .

For each of the two cases, a simple but quite lengthy calculation leads to the following expression for a λ -dependent contribution to the inner radiative correction:

Case I:

$$\Delta_{inner} = \lambda \frac{\alpha}{2\pi} \frac{1}{\xi_0^2} \left((\bar{\mu}_p - \bar{\mu}_n)(\xi_0 - 1) \ln(1 - \xi_0) + 2[\xi_0 - 2 - \ln(1 - \xi_0)] \right), \quad (7)$$

Case II:

$$\Delta_{inner} = \lambda \frac{\alpha}{2\pi} \left((\bar{\mu}_p - \bar{\mu}_n) \left(\frac{13}{12} - \xi \ln \xi \right) + \frac{7}{6} + 2\xi \ln \xi + \frac{1}{\xi^2} + \frac{3}{4} \ln \xi - \frac{1}{4\xi} \ln \xi \right), \quad (8)$$

where $\bar{\mu}_p$ and $\bar{\mu}_n$ are the proton and neutron anomalous magnetic moments, $\xi_0 = M^2/\Lambda_0^2$ and $\xi = M^2/\Lambda^2$. We have checked that the two rather different phenomenological approaches used here give numerically similar results within reasonable variations of the model parameters, Λ_0 and Λ . Even in the absence of these cut-off parameters (point-like nucleons with a cut-off at the scale of the vector boson mass) the calculated inner corrections does not exceed the values in Eqs.(7) and (8) by more than an order of magnitude. This indicates a certain degree of stability of our results.

In the above we concentrated on the inner corrections. We now turn our attention to the outer correction, which is defined as an E_m -dependent correction. As is well known [3], if one only retains terms of $O(\alpha)$, ignoring terms of $O(\alpha E/M)$ and $O(\alpha(E/M) \ln(M/E))$ (E = electron energy), then the outer correction becomes model-independent and hence also λ -independent. The term $\bar{g}(E_m)$ in Eq. (5) represents this model-independent contribution. However, at the level of precision we are interested in, it becomes necessary to retain terms of $O(\alpha E/M)$ and $O(\alpha(E/M) \ln(M/E))$, and these terms lead to a λ -dependent outer correction. We write a generic form of the λ -dependent outer correction as

$$\Delta_{outer} = \lambda \frac{\alpha}{2\pi} f(E_m). \quad (9)$$

The explicit forms of $f(E_m)$, which depend on the models used, are very lengthy and will be given elsewhere. We remark that the separation of the radiative corrections into the terms in Eq.(5) is specific to the quark model, and therefore it is not possible to establish termwise correspondence between the terms in Eq.(5) and those in Eqs.(7) - (9).

We denote by Δ_λ the leading-order λ -dependent radiative correction, which is the sum of the inner correction in Eq. (7) or Eq. (8), and the outer correction in Eq. (9):

$$\Delta_\lambda \equiv \Delta_{inner} + \Delta_{outer} \equiv \lambda \cdot \frac{\alpha}{2\pi} J. \quad (10)$$

Here J contains information about the nucleon structure including its weak and electromagnetic form factors. Assuming that the cut-off parameters in Eqs.(7)-(9) are controlled by a strong interactions scale ($\Lambda_0 \sim \Lambda \sim M$) and that a characteristic energy of nuclear β -decay is $E_m \sim 5$ MeV, the value of J turns out to be typically $5 \sim 25$. In the subsequent numerics, we use $J = 15 \pm 10$.

Since we have a general expression (within our schematic model) for the λ -dependent part of the $O(\alpha)$ radiative corrections, we may use it to study the influences of the in-medium modification of λ . We expect that, if a description based on the nucleon weak current achieves a sufficient level of precision, then the result corresponding to the case with λ *fixed at the free nucleon value* should essentially reproduce the results of the existing analyses

of the relevant observables based on Eq.(5). Then we may assume that small perturbative changes in the λ -dependent radiative correction due to in-medium modification of λ can be estimated with the present schematic model. We use Eq.(10) in this limited context.

We introduce the in-medium renormalization parameter ρ as $\rho \equiv \lambda^*/\lambda$. For a wide range of the periodic table, a typical value of ρ is $\rho \sim 0.8$ [10,11]. For our present purpose, we fix ρ at 0.8. Then, by taking the difference of Δ_λ , Eq. (10), between the nuclear and free cases, we obtain

$$\Delta \equiv \Delta_\lambda - \Delta_{\lambda^*} = \lambda(1 - \rho)\frac{\alpha}{2\pi}J. \quad (11)$$

This leads to a corrected value [5] of the CKM matrix element V_{ud}

$$V_{ud} \rightarrow V_{ud}(1 + \Delta/2), \quad (12)$$

and, since the probability of neutron β -decay is proportional to the $|V_{ud}|^2$, to a corrected expression for the neutron life time

$$\tau_n = \frac{2(Ft)}{f_R \ln 2} \frac{(1 - \Delta)}{(1 + 3\lambda_\tau^2)}. \quad (13)$$

Here F and f_R are nuclear and neutron phase space factors [5,12]. Eqs.(12) and (13) summarize how the medium dependence of λ encoded in Δ can change the deduction of V_{ud} and the analysis of τ_n . For the choice of the values of parameters explained above, we have $\Delta = 0.0044 \pm 0.0029$.

The new formula Eq.(12) changes the value of V_{ud} from $V_{ud} = 0.9740 \pm 0.0005$ to $V_{ud} = 0.9761 \pm 0.0015$. The error for the CKM matrix element has been estimated simply as the sum of the squared errors, one coming from Eq.(1) and the other coming from the allowed range of J . The new value of the CKM matrix element in turn leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0013 \pm 0.0031 \quad (14)$$

in good agreement with the unitarity condition. Moreover, the use of Eq. (13) with $\Delta = 0.0044 \pm 0.0029$ decreases λ_τ from the value in Eq.(4) to $\lambda_\tau = 1.2655 \pm 0.0034$. This new value agrees with λ_A , Eq.(3) within the error bars.

Thus, we have illustrated the possibility that the inclusion of the effect of the in-medium modification of G_A in the entire radiative correction (not only in the C term) is capable of removing the two outstanding problems regarding the basic coupling constants G_V and G_A . Our present treatment is admittedly very schematic and subject to various improvements, and the particular numerical successes reported above are simply indication of the possible effect. However, we believe that our calculation does indicate the importance of the new type of radiative correction considered here and that future analyses of the $0^+ \rightarrow 0^+$ β -transitions need to include the effects discussed in this article.

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